Inside Debt, Aggregate Demand, and the Cambridge Theory of Distribution

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I Introduction

The Kalecki (1942)-Kaldor (1955/56) theory of aggregate demand and income distribution represents an enduring core of Post Keynesian analysis, which shows how the distribution of income is important for the level of aggregate demand, and how the distribution of income is itself determined by the savings propensity of the capitalist class.[1] Another feature of Post Keynesian analysis has been emphasis on the inability of the price system to ensure full-employment, a key reason for which, is the existence of inside debt (Dutt, 1986: Palley, 1991). This line of reasoning borrows from Fischer's (1933) debt-deflation arguement, which maintains that decreases in the price level may actually reduce consumption and aggregate demand because they increase the burden of existing inside debts.

The current paper seeks to synthesize the above elements of Post Keynesian analysis by introducing inside debt into the Cambridge theory of income distribution. The paper begins with a Kaleckian model of aggregate demand that incorporates inside debt, and this model is then used to analyse the determination of income distribution in the presence of inside debt service papyments. The model gives rise to a number of innovations which include (i) the introduction of a generational structure and population growth into the Post Keynesian model of aggregate demand, and (ii) a modification of the Cambridge theorem which shows that borrowing by workers affects the profit rate and profit share.

These results contrast with Baranzini's (1982) examination of the implications of including life-cycle savings considerations (with and without bequests) for the Cambridge theorem. In that paper Baranzini showed that life-cycle considerations left the Cambridge theorem intact, a conclusion which contrasts with the current paper. The reason for this difference is the recognition of the effects of "inter-class" income transfers resulting from inside borrowing by workers. This feature was absent in Baranzini's formulation in which life-cycle borrowing only caused "intra-class" transfers so that there was no redistribution of income across classes as a result of inside debt.

II A model of aggregate demand with inside debt

This section develops a Kaleckian model of aggregate demand that includes inside debt and population growth. As is standard in such models, there are two classes -- workers and capitalists.[2] For simplicity, only capitalists are assumed to save, while only workers borrow. In the model that is developed below both workers and capitalists have well-defined life-time consumption profiles that obey lifetime budget constraints, and in this sense the model is inter-temporal in character. However, these consumption profiles are not derived from the solution of life-cycle utility maximization programs; consequently, there are no inter-temporal substitution effects arising from changes in interest rates. If such effects were present, then workers' marginal propensity to borrow and capitalist' marginal propensity to consume would depend on the level of interest rates. However, in the absence of particular assumptions about the functional form of the utility function, the signing of such effects would be ambiguous because of offsetting income and substitution effects.

Aggregate demand consists of demand from the worker class, demand from the capitalist class, and exogenous investment, so that

(1) $AD = AD_w + AD_c + I$

where AD = aggregate demand

AD_w = demand by workers

AD_c = demand by capitalists

I = level of investment spending

The presence of inside debt introduces a number of important considerations which include the aggregate demand effects of financing new debt, and the aggregate demand effects of paying back existing debt. In addition, the presence of debt and the requirement of repayment forces the recognition of life-cycle and generational structures, and this gives rise to aggregate demand effects arising from behavioral differences between young and old generations. The existence of inside debt therefore has major ramifications for the specification of aggregate demand.

The aggregate demand of workers is given by

(2) $AD_w = (w + b)N_1 + (w - (1 + i)b)N_2$

where w = wage rate

b = borrowing per worker

i = interest rate on debt

N_1 = number of employed young workers

N_2 = number of employed old workers

Equation (2) has workers borrowing when young, and paying back when old. Initially borrowing is assumed to be independent of the interest rate, but this is relaxed later. It is also assumed that $w - (1 + i)b > 0$, so that older workers are able to pay back their debts.[3] The relationship between the number of employed young and old workers is determined by the rate of population growth, and given by

(3) $N_1 = (1 + g)N_2$

where g = the rate of population growth. The total number of employed workers is

(4) $N = N_1 + N_2$

The aggregate demand of capitalists is given by

(5) $AD_c = c_1(P + ibN_2)/2 + c_2(P + ibN_2)/2$

where c_1 = propensity to consume of young capitalists: $0 < c_1 < 1$

c_2 = propensity to consume old capitalists: $0 < c_2 < 1$

P = aggregate profits.

Capitalists' income consists of total profits and interest on debts previously incurred by older workers, and this is divided between young and old capitalists.[4]

The specification of capitalists' demand raises important issues about debt financing and bequests. Per (5), it is only the interest payments on inside debt that affect capitalist demand. This treatment reflects the assumption that worker borrowing is financed through a "credit money banking system", in which banks pay out all interest income to capitalists who own the banks. In such a system, new borrowing and repayment of existing borrowing result in no transfers between workers and capitalists. Borrowing generates new loans, while repayment extinguishes existing loans: it is only the interest payments that result in transfers. Such a system contrasts with a "loanable funds" approach to credit in which borrowing for consumption by workers must be matched by a reduction in consumption by capitalists. Analogously, repayments of borrowings reverse the transfer between workers and capitalists. Per the loanable funds vision, loan creation and extinction both give rise to income transfers between capitalists and workers.

A second feature of equation (5) concerns bequests, and the distribution of profits between young and old capitalists. For simplicity, each generation is assumed to hold half of the total stock of wealth, which in turn entitles each generation to half of profits and half of interest payments. This enormously simplifies the algebra of the comparative statics, without doing violence to the economic insights of the model: the appendix derives the strict income shares of capitalists based on differential generational propensities to consume, and population growth amongst capitalists. The source of young capitalists' income is bequests from older capitalists who have just died, and who are assumed to bequest all their wealth to the young generation of capitalists.[5]

Finally, closing the model requires the following equations

(6) $P = y - wN$

(7) $y = aN$

(8) $y = AD$

where y = output

a = coefficient of the production function

Equation (6) is the definition of profits. Equation (7) is the production function. Equation (8) is the goods market clearing condition. By appropriate substitution, the model can be reduced to a single equation in N_2 given by

(9) $N_2 = I/D$

where $D = [(a-w)(2+g)(1 - (c_1+c_2)/2) - (c_1+c_2)ib/2 - (g-i)b] > 0$

Differentiating (9) with respect to the exogenous variables yields

$dN_2/dI = 1/D > 0$

$dN_2/dc_1 = I[(a-w)(2+g) + ib]/2D^2 > 0$

$dN_2/dw = I[(2+g)(1 - (c_1+c_2)/2]/D^2 > 0$

$dN_2/da = -I[(2+g)(1 - (c_1+c_2)/2]/D^2 < 0$

$dN_2/dg = -I[(a-w)(1 - (c_1+c_2)/2) - b]/D^2 \gtrless 0$

if $b \gtrless (a-w)(1 - (c_1+c_2)/2)$

$dN_2/db = -I[-(c_1+c_2)i/2 - (g - i)]/D^2 \gtrless 0$

$$\text{if } g + (c_1 + c_2)i/2 \ {}^{>}_{<}\ i$$

$$dN_2/di = -I[b(1 - (c_1+c_2)/2)]/D^2 < 0$$

Increases in the level of autonomous investment spending, capitalists' propensity to consume, and the wage rate all increase employment. Increases in the level of labor productivity, decrease employment. These results are standard within the Kaleckian model. The novel results concern the effect of the rate of worker population growth, and the level of per capita worker borrowing.

Increases in the rate of population growth are expansionary if the borrowing per additional worker exceeds the savings out of the profit produced by an additional worker. This is a form of "demand leading" growth, whereby borrowing serves to absorb the savings of capitalists out of additional income. Such an effect of population growth is consistent with Keynes' observations in his 1937 address to the Eugenics Society on the economic consequences of a declining population (Keynes, 1937). However, whereas Keynes emphasized the effect of population growth on the demand for new capital, the current focus is its effect on consumer borrowing: this provides an additional source of increment in aggregate demand necessary to employ a growing population.[6]

Increases in the level of borrowing are expansionary if the direct demand effect plus capitalists' spending out of induced interest income exceeds the lost worker consumption spending arising from the transfer of additional interest needed to service the extra debt. Lastly, increases in the interest rate decrease employment since aggregate demand is reduced as a consequence of larger transfers of income from workers to capitalists. This effect is supplementary to any effect that interest rates may have on investment spending.

In standard life-cycle utility maximization models of household choice, consumption decisions are commonly assumed to depend negatively on the level of interest rates. If capitalists' marginal propensities to consume (c_1, c_2) and workers' demand for borrowings (b) are assumed to be negative functions of the interest rate, then inspection of equations (2) and (5) reveals that the effect of interest rates on aggregate demand is ambiguous. With regard to workers' aggregate demand, higher interest rates reduce both borrowing and interest payments, so that the net effect is ambiguous: with regard to capitalists' aggregate demand, higher interest rates reduce the marginal propensities to consume, but the effect on debt service income is ambiguous.

The above model may be refined to include additional details regarding the determination of worker borrowing. Thus, worker borrowing may be determined according to

$$(10)\quad (1 + i)b = vw$$

where v = coefficient of debt plus debt service to income ratio. Per (10), worker borrowing is limited by the required debt plus debt service to wage income ratio.[7] Rearranging (10), and substituting in (9) yields

$$(11)\quad N_2 = I/[(a-w)(2+g)(1 - (c_1+c_2)/2) - (c_1+c_2)ivw/2(1+i) - (g-i)vw/(1+i)]$$

Differentiating with respect to w yields

$$dN_2/dw = I[(2+g)(1 - (c_1+c_2)/2) + (c_1+c_2)iv/2(1+i) + (g-i)v/(1+i)]/D^2$$

Now wage changes have additional demand implications because of their impact on worker borrowing. There is a positive direct effect on young worker consumption, a positive indirect effect on capitalist consumption arising from increased debt service income, and a negative effect on older worker consumption owing to larger debt service and repayments.

III Inside debt and the Cambridge theorem

The above Kaleckian style model of aggregate demand can now be used to examine the implications of inside debt for the Cambridge theory of income distribution as developed by Kaldor (1955/56), and extended by Pasinetti (1961/62). The one significant change from the above, which is added for purposes of greater generality, is that workers as a class are now assumed to have positive savings. This means that workers own part of the capital stock, and in accordance with the standard Cambridge assumption, workers' share of the capital stock is equal to their share of total saving.[8]

The fact that workers own part of the capital stock imposes the following adding up constraints

(12) $P_1 + P_2 + P_w = P$

(13) $K_1 + K_2 + K_w = K$

(14) $S_j/S = K_j/K = P_j/P = z_j \qquad 0 < z_j < 1 \qquad j = 1,2,w$

where P_1 = profits paid to young capitalists

P_2 = profits paid to old capitalists

P_w = profits paid to workers

P = total profits

K_1 = capital owned by young capitalists

K_2 = capital owned by old capitalists

K_w = capital owned by workers

S_j = total savings of the j^{th} class

S = total savings

The central organizing relation of the Cambridge theorem is the requirement that savings by capitalists equal the share of investment that capitalists must fund, where this share is equal to their share of profits. This implies

(15) $(z_1 + z_2)I = S_1 + S_2$

where S_1 = saving by young capitalists

S_2 = saving by old capitalists

The savings functions for capitalists are given by

(16) $S_j = s_j z_j(P + iB)$ $\qquad j = 1, 2$

where $s_i = (1 - c_i)$ = propensity to save of i^{th} generation capitalists

B = existing stock of inside debt

The logic of (16) is that each generation of capitalists saves a fraction, s_j, out of its share, z_j, of total profit and interest income, P + iB. Substituting (16) into (15) yields

(17) $(z_1 + z_2)I = (s_1z_1 + s_2z_2)(P + iB)$

The central proposition of the Cambridge theorem is that in the long-run workers' propensity to save does not matter for the determination of the profit share or profit rate:

"This is the most striking result of our analysis. It means that, in the long-run, workers' propensity to save, though influencing the distribution of income between capitalists and workers does not influence the distribution of income between profits and wages. Nor does it have any influence whatsoever on the rate of profit (Pasinetti, 1961/62, p.272)"

The significant feature about the introduction of inside debt is that this proposition needs to be modified to take account of workers' propensity to borrow. To see this we need to slightly modify the model presented in section II, and allow the co-existence of borrowing and saving by workers.

Let q = the proportion of young workers who incur debt where $0 < q < 1$. It is assumed that workers who borrow have no saving, so that if q = 1, then all young generation workers are borrowers and there is no saving by that group. Moreover, borrowing by individual young workers continues to be governed by equation (10), so that the stock of outstanding debt is given by

(18) $B = vwqN_2/(1 + i)$

where v = young workers' propensity to borrow

q = proportion of young workers who borrow

Substituting (18) into (17), and manipulating appropriately yields

(19.a) $P/K = (z_1 + z_2)I/(s_1z_1 + s_2z_2)K - ivwqN/(1+i)(2+g)K$

(19.b) $P/y = (z_1 + z_2)I/(s_1z_1 + s_2z_2)y - ivwqN/(1+i)(2+g)y$

These are the familiar Cambridge conditions, modified to capture generational differences in the behavior of capitalists, and augmented by a term to capture the impact of inside debt. If young capitalists and old capitalists have the same savings propensity so that $s_1 = s_2 = s$, then (19.a) and (19.b) simplify to

(19.c) $P/K = I/sK - ivwqN/(1+i)(2+g)K$

(19.d) $P/y = I/sy - ivwqN/(1+i)(2+g)y$

If no young workers borrow, so that q = 0, then these conditions simplify to

(19.e) $P/K = I/sK$

(19.f) $P/y = I/sy$

which are the conditions derived by Pasinetti (1961/62, p.272).

In long-run steady-state the rate of interest is equal to the profit rate, which implies that[9]

(20) $P/K = i$

Moreover, the rate of growth of the capital stock, I/K, is equal to the rate of growth of the workforce so that

(21) $I/K = g$

The capital:output, labor:output, and capital:labor ratios are also all constant. Substituting (21) into (19.a) and (19.b) yields

(22.a) $P/K = (z_1 + z_2)g/(s_1z_1 + s_2z_2) - ivwqN/(1+i)(2+g)K$

(22.b) $P/y = (z_1 + z_2)gK/(s_1z_1 + s_2z_2)y - ivwqN/(1+i)(2+g)y$

Equations (20) and (22.a) can then be jointly solved for the equilibrium profit rate, while equations (20) and (22.b) can be jointly solved for the equilibrium profit share.

The graphical solution for the profit rate is shown in figure (1). Equation (22.a) is convex to the origin when drawn in [P/K, i] space. The same is also true of equation (22.b) when drawn in [P/y, i] space. The significant feature about the solution is that it depends on the parameters g, v, and q. Moreover, the parameters v and q relate to the consumption behavior of the young generation workers, and this implies that the consumption behavior of workers (and therefore their saving behavior as a class) is relevant to determination of the profit rate and profit share.[10]

The graphical analogue of the model given by figure (1) can now be used to solve for its comparative static properties. Increases in g, the natural rate of growth, shift the P/K schedule up. This results in an increase in the steady-state profit rate. It also results in an increase in the steady-state profit share. The logic is that population growth raises investment per equation (21), and this requires higher profits for capitalists to fund their share of investment: this is an outcome consistent with Keynes' (1937) argument about the effect of population growth on capital accumulation and aggregate demand.

Increases in the propensity to borrow, v, and the proportion of young workers who borrow, q, shift the P/K schedule down. This results in a reduction of the steady-state profit rate, and it also reduces the steady state profit share. The logic is that increased inside debt raises the income capitalists receive in the form of debt service. This means that the level of profits must adjust downward to ensure that capitalist saving out of total income (profits plus debt service) remains equal to the share of investment which they are required to fund. Increased borrowing and the development of mass consumption financed by expanding worker borrowing is therefore good for the level of aggregate demand, but it is ultimately bad for the profit rate and profit share. This effect may explain some of the apparent secular decline in the profit rate. Presumably, the economic mechanism is a reduction in margins brought about by reduced aggregate demand: increased stocks of inside debt result in increased transfers of income from workers to capitalists, which induces a reduction in the profit rate and interest rate in order to preserve equilibrium between savings and investment.

IV Conclusion

This paper presented a Kaleckian model of aggregate demand that included inside debt and a generational structure that distinguished betwen young and old workers and capitalists. The model was then used to show how worker borrowing and population growth served to increase aggregate demand. This inclusion of the aggregate demand effects of population growth links with observations made by Keynes in 1937 on the same issue. However, whereas Keynes emphasized the investment demand effects of population growth, the current paper emphasized their effect on aggregate consumption arising from greater consumer borrowing.

The model of aggregate demand was then used to examine the Cambridge theory of distribution. The key finding was that inside debt invalidates the Cambridge claim that the steady-state profit rate and profit share are independent of worker consumption behavior. Instead, increased borrowing by young generation workers serves to reduce both the profit rate and profit share. The logic was that increased borrowing generated a higher debt sevice income for capitalists, and this called for lower profits to ensure balance between capitalists' total incomes and their investment funding requirement.

Appendix

This appendix derives the strict division of profit and interest income amongst young and old capitalists. Following Pasinetti (1961/62), ownership shares are proportional to relative savings rates. The young generation of capitalists derive their income from bequests, so that their ownership share is based on the savings rate of old capitalists: the old generation of capitalists derive their income from wealth they accumulated when young, so that their ownership share is based on the savings rate of young capitalists.

Assuming no population growth amongst capitalists, the respective shares are

(A.1) $z_1 = s_2/(s_1 + s_2) = (1 - c_1)/[(1 - c_1) + (1 - c_2)]$

(A.2) $z_2 = s_1/(s_1 + s_2) = (1 - c_2)/[(1 - c_1) + (1 - c_2)]$

where z_i = ownership share of ith generation: i = 1,2
s_i = propensity to save of the ith generation
c_i = propensity to consume of ith generation

If there is population growth amongst capitalists the ownership shares are derived using population weighted average propensities to save so that

(A.3) $z_1 = s_2/[s_1/(1+g_c) + s_2]$

(A.4) $z_2 = [s_1/(1 + g_c)]/[s_1/(1 + g_c) + s_2]$

where g_c = rate of growth of capitalist population.

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Abstract

This paper presents a Kaleckian model of aggregate demand with inside debt and a generational structure. The model shows how worker borrowing and population growth impact the level of aggregate demand. It also re-examines the Cambridge theorem, and shows that the introduction of inside debt arising from worker borrowing means that the rate of profit and profit share are no longer independent of worker consumption behavior because increased worker borrowing increases capitalists' debt service income. This necessitates a decline in profits to maintain balance between capitalists' savings and their investment funding requirement.

Keywords: Inside debt, aggregate demand, population growth, Cambridge Theorem.

JEL ref.: E0, E1

Profit
Rate

$P/K=i$

$P/K=(z_1+z_2)I/(s_1z_1+s_2z_2)$

$-ivwqN/(1+i)(2+g)K$

45^0

Interest Rate

Figure (1): Shows the determination of the equilibrium profit rate and interest rate.

1..Kaldor (1955/56) derived a special case of the Cambridge theorem of income distribution based upon the assumption that workers had no savings. Pasinetti (1961/62) showed that the Cambridge theorem continued to hold for the general case in which workers had positive savings.

2..The purpose of the Kaleckian specification is to introduce a distinction between the propensities to consume out of wage and profit income. There are many justifications for this set-up. One is that there are literally two classes of agents, and these classes have different behavioral propensities. A second justification is that agents psychologically treat wage and profit income differently, saving more out of profit income. A third justification is that profit income predominantly goes to high income households, and these households have a lower propensity to consume: consequently, the aggregate average propensity to consume out of profits is lower than that out of wages.

3.. The representative worker's consumption plan and budget constraint are given by $C_1 = w + b$: $C_2 = w - (1+i)b$: $C_1 + C_2/(1+i) = w + w/(1+i)$ Workers therefore have an explicit life-cycle consumption plan. This plan can be rendered consistent with the conventional life-cycle utility maximization model by assuming that workers are liquidity constrained and can only borrow b when young. If workers were unconstrained, then b would be a function of the interest rate.

4..Assuming the population of capitalists is also growing at rate g, the ratio of per capita consumption of young and old captalists is given by $c_1/c_2(1+g)$. To place capitalists' choice of consumption plan in an explicit life-cycle utility maximization framework would necessitate making the parameter's c_1 and c_2 functions of the interest rate and level of income. In the current model, capitalists' savings in period 1 increase with the interest rate, which is consistent with an argument made by Bear (1961) regarding the dominance of the substitution effect.

5..The model effectively embodies a "permanent income" approach to wealth. Capitalist income derives from holdings of capital and inside debt, and by fully accounting for income from these sources, one has accounted for wealth. In principle there could also be income transfers between young and old capitalists arising from transactions in non-productive wealth. For instance, young capitalists might use some of their income to purchase old master paintings, from old capitalists: alternatively, old capitalists could purchase the paintings inherited by the young.

6..Keynes' (1937) argument regarding the effect of population growth on the demand for new capital can be accommodated in the current model by making investment a positive function of population growth.

7..The representative worker's consumption plan and budget constraint are $C_1 = w + b$: $C_2 = w - (1+i)b$: $C_1 + C_2/(1+i) = w + w/(1 + i)$. From equation (10) worker borrowing is $b = vw/(1+i)$, so that $C_1 = w(1-v/(1+i))$ and $C_2 = w(1-v)$. Once again there is an explicit lifetime consumption profile, and this profile can be rendered consistent with a utility maximizing life-cycle model by assuming workers face an exogenously given liquidity constraint of $vw/(1+i)$.

8..There are two ways of explaining the co-existence of worker borrowing and worker ownership of the capital stock. First, there may be some worker households that borrow and have no saving, while other worker households have saving and don't borrow. Second, workers may in the second period of their lives save in excess of their debt obligations, and this excess is then transferred as a bequest to the young generation of workers. Thus, young workers spend their bequest as well as borrowing, so that workers as a class are net debtors.

9..This condition is used by Dalziel (1991) in his analysis of the Cambridge theorem in the presence of government debt. This closure differs from that used by Baranzini (1982) in which the interest rate is determined so as to clear the loanable funds market.

10..If q = 1, then the entire young generation of workers are borrowers and have no savings. However, workers as a class may still be entitled to a share of profits if older workers save in excess of the debt obligations they incurred when young.