The Stock Market and Investment: Another look at the micro-foundations of q Theory

Abstract

This paper introduces a novel distinction between real \( q \) and financial \( q \). The paper examines three versions of financial \( q \) developed by Brainard and Tobin, Minsky, and Hayashi. These theories differ regarding the nature of stock market price determination and their use of marginal productivity theory. It is shown that non-profit maximizing behavior by managers does not invalidate \( q \) theory. It is also shown that if managers and shareholders have different profit expectations, this leads to an equilibrium value of \( q \) that differs from unity. Lastly, the implicit claims in \( q \) theory regarding the efficient role of stock markets as regulators of capital accumulation are shown to depend on assumptions about stockholder behavior.

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I The implications of stock market $q$ for policy

The monetary transmission mechanism remains an enduring area of interest in monetary macroeconomics. Macroeconomic models have traditionally emphasized the interest rate channel which affects investment spending by establishing a hurdle rate of return for the marginal efficiency of investment. However, this traditional channel has been replaced increasingly by Brainard and Tobin's (1968, 1977) $q$ theory of investment. Their approach emphasizes equity prices, and shifts attention away from the bond and money markets toward equity markets. In place of interest rates, equity prices become the main channel whereby monetary policy affects investment spending (Blanchard, 1981).\(^1\)

The $q$ approach to the transmission mechanism increases the macroeconomic significance of stock markets which now take on an important role in managing the process of capital accumulation. This argument has taken on heightened policy significance owing to the debate over privatization of state funded old age pension schemes. Privatization would likely increase saving in the form of equities, thereby increasing equity prices and $q$. If $q$ theory holds, this would raise investment spending and capital accumulation, thereby helping address the problem of supporting an aged population.

A second reason why $q$ theory has taken on increased policy significance is the shift within the U.S. to budget surplus. Fiscal conservatives argue that a reduced national debt will increase investment because reduced supplies of debt will lower interest rates and have a portfolio knock-

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1. Both $q$ and the conventional Keynesian interest rate mechanism share a reliance on asset market "price" signals as the means for affecting real economic activity. This contrasts with theories emphasizing "quantity" effects, such as the credit rationing approach (Stiglitz and Weiss, 1981). In addition to affecting investment spending through $q$, equity prices also affect consumption spending through their impact on household wealth. This wealth effect is separable and distinct.
on effect that raises equity prices and $q$. Finally, a third policy development concerns the fostering of equity markets in developing countries. If $q$ matters for investment, then developing countries should be encouraged to promote local stock markets, and this provides a justification for privatization in order to create widespread equity ownership.

For all of the above reasons, $q$ theory of investment has taken on greater policy significance. This paper analyzes the theoretical foundations of $q$’s claimed link between equity markets and investment spending. It clarifies the differences between alternative constructions of $q$, and shows that these rest on (i) different interpretations of the efficiency of stock markets as regulators of capital accumulation, and (ii) differences regarding firms' objectives, competitive conditions, and relations between managers and owners.

II Conventional versus $q$ theoretic specifications of investment

Conventional macroeconomics links the financial and real sectors through the investment function, with the financial sector determining the general level of interest rates, and interest rates affecting real investment spending. The conventional investment function is given by

\[ I = I(r, MEK, ...) \quad I_1 < 0, I_2 > 0 \]

where $I$ = investment spending; $r$ = real interest rate; $MEK$ = marginal efficiency of capital.

This conventional investment function contrasts with a $q$ theoretic specification given by

\[ I = I(q) \quad I_q > 0, I(1) = dK \]

(3) $q = P_e/P_K$

(4) $P_e = P_EE/K$

where $P_e$ = shadow equity market price of a unit of capital; $P_K$ = current cost of a unit of capital; $P_E = unit price of equities; E = number of equities in issue; $K$ = capital stock; $d$ = rate of
depreciation. $P_e$ is the implicit value of capital established through the stock market, and it is obtained by dividing the total value of equities by the existing capital stock. Substituting (4) into (3) yields

\[ q = \frac{P_E}{P_K} \]

$q$ is therefore the ratio of the stock market valuation of the corporate sector ($P_E$) to the current replacement cost of physical capital owned by the corporate sector ($P_K$). According to $q$ theory, an increase in stock market prices raises $P_e$ and $q$, which then increases investment spending.

**III A new distinction: real versus financial $q$**

One possible interpretation of $q$'s economic logic is that it represents an arbitrage theory of investment built upon the "law of one price". This explanation is suggested by Brainard and Tobin (1977), who attribute it to Keynes. Of this arbitrage mechanism Keynes wrote

"[The] daily revaluations of the stock exchange, though they are primarily made to facilitate transfers of old investments between one individual and another, inevitably exert a decisive influence on the rate of current investment. For there is no sense building up a new enterprise at a greater cost than that at which a similar existing enterprise can be purchased: whilst there is an inducement to spend on a new project what may seem an extravagant sum, if it can be floated off on the Stock Exchange at an immediate profit (The General Theory, p.151)."

If new and existing capital are perfect substitutes, firms wishing to acquire additional capital will have an incentive to purchase new capital when $q > 1$ as it costs less than existing capital. Suppliers of capital goods will also have an incentive to produce and sell more, so as to exploit the premium embedded in the current market price. By analogous reasoning, if $q < 1$ the incentives are reversed.

The plausibility of this "arbitrage" interpretation depends on whether stock markets represent implicit second hand markets for physical capital. If they do, an increase in stock market prices amounts to an increase in the price of existing capital, and it provides an incentive for firms to
invest in new capital rather than purchasing existing capital.

Such a description of the relationship between asset prices and investment spending makes sense for assets such as structures. When house and building prices are above replacement cost (i.e. housing $q > 1$), builders have an incentive to build and sell more, and buyers have an incentive to buy new houses. New houses are relatively close substitutes for existing houses, and there are well established markets for existing houses. However, application of this logic to equity markets implies that (i) equities are simply a "veil" over a market for existing capital in which an increase in household demand for equities represents an increase in household demand for holdings of physical capital, and (ii) managers of firms alter their investment spending plans in response to changed equity prices.2

The above considerations suggest distinguishing between "real" $q$ theory and "financial" $q$ theory. Real $q$ applies where there are well defined markets in which new and existing capital goods are close substitutes and traded side-by-side: it operates through goods market arbitrage predicated on the law of one price. Financial $q$ works through a different mechanism.3

IV Competing interpretations of financial $q$

Financial $q$ works through the stock market. Understanding the impact of stock prices on

2. A third issue concerns the degree of substitutability between new and existing capital. However, this not the key issue, and $q$ theory addresses this matter by distinguishing between "marginal" (new capital) and "average" (existing capital) $q$. If new and existing capital are perfect substitutes, then marginal and average $q$ are the same.

3. Brainard and Tobin (1990, p.546-7) again make mention of the arbitrage channel, writing “High prices for resale of houses or cars surely signal good opportunities for builders or automobile manufacturers. The $q$ ratio applies that common sense idea to producers’ capital formation. But used capital goods markets are not reliable sources of values because of the specificities, complimentarities, and irreversibilities of most producers’ structures and equipment. The stock and bond markets provide valuations of whole businesses.” These comments suggest that the arbitrage channel is not the intended basis of $q$ theory’s equity market - investment link.
investment raises questions regarding the shareholder - stock market - manager nexus.

One issue concerns how equity prices are determined. Here, there are two different analytic traditions. The first, emphasized by Keynes, sees equity prices as embodying significant sociological, and possibly irrational, influences. It is captured in Keynes' (1936, p.156) metaphor of professional investing being akin to a beauty contest. The second tradition is the modern neoclassical theory of equity prices which sees them as reflecting the "fundamental" value of firms. This fundamental value is the expected present discounted value of firms' streams of current and future profits, which is given by

$$ PE = \int_{t=0}^{\infty} V^E(t) e^{-d(t)t} dt / E $$

where $V^E(t)$ = expected level of firms' profits at time $t$ conditional on information available at time $t = 0$; $d(t)$ = wealth holders discount rate at time $t$. According to this second view, wealth holders engage in rational calculations that convert their expectations of future profits into a net present value equivalent.

Independent of their process of determination, stock prices matter in a $q$ framework because they implicitly determine firms' cost of equity capital, which is given by

$$ r_E = V^E / PE $$

where $r_E$ = the cost of equity capital; $V^E$ = shareholders' expectation of the level of firms' profits (which are now assumed constant for simplicity). Thus, higher stock prices lower the cost of equity capital.

*Brainard and Tobin: strong $q$.*

Given the above, we are now in a position to examine the economic logic of alternative constructions of $q$. Brainard and Tobin (1968, 1977 -- henceforth B-T) can be identified with the
"strong" version of $q$ in which equity markets affect the cost of equity capital and send coherent signals about the marginal efficiency of capital. For B-T, demand for equities represents a portfolio demand for physical capital by wealth holders. B-T therefore see equities as a "veil" through which shareholders see to the underlying capital assets owned by firms. Moreover, according to B-T equity prices are determined by the fundamental valuation model described in (6), and equity prices therefore reveal an implicit shareholder required rate of return that is determined by equation (7). This required return establishes the cost of equity capital to managers.

Putting these pieces together, increased demand for physical capital by portfolio holders raises equity prices which lowers the cost of equity capital, thereby giving managers an incentive to undertake more investment.

The transmission mechanism from equity markets to investment spending works via managers' project net present value (NPV) calculations. Managers undertake investment as long as projects have positive NPV, which requires satisfying the condition

$$(8) \text{NPV} = \frac{R}{r_E} - P_K > 0$$

where $R =$ managers' expectations of the additional revenues from an extra unit of capital. The term $R/r_E$ represents the contribution of the marginal project evaluated at the current cost of equity capital. If the marginal return to capital is the same as the average return and managers have the same expectations as shareholders, then $R = V/K = V^E/K$. Using this condition in combination with equation (7), means that (8) can be expressed as

$$(9) \text{NPV} = \frac{P_E}{P_K} - 1 > 0$$

$$= q - 1 > 0$$

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4. This is tantamount to the assumption that marginal $q$ equals average $q$. 
Thus, according to B-T, the statement that managers only undertake projects with positive net present values reduces to the claim that managers only undertake projects with a $q$ value greater than one.

From the B-T perspective, both stock markets and higher stock prices are "good". Stock markets allow wealth holders to communicate to firms their demands for physical capital, and firms respond by accumulating the amount desired. Higher stock prices raise investment and capital accumulation by lowering firms' cost of capital.

$q$ is a particularly useful empirical indicator since it is a "composite" variable that combines "cost of capital" and "marginal efficiency of investment" considerations. This composite character is illustrated in Figure 1. The cost of equity capital reflects both wealth holders' expectations of future earnings and their willingness to hold capital since these factors are embodied in equity prices. As a result $q$ embodies both real and financial considerations affecting investment, and it is this composite character that recommends $q$ for empirical purposes of estimating investment spending.

Lastly, in deriving (9) it was assumed that managers' expectations are the same as shareholders' expectations. Section IV examines what happens when this is not the case, and shows that the $q$ transmission mechanism still holds but the equilibrium value of $q$ is changed.

*Minsky: weak $q$.*

Minsky (1975, 1986) can be identified with a "weak" version of $q$ in which equity markets affect the cost of capital, but send no coherent signals about the marginal efficiency of capital. Minsky shares the Keynes/B-T view that equity markets act as surrogate markets for existing capital. Thus, Minsky writes:

"There are really two systems of prices in a capitalist economy -- one for current output and the
other for capital assets. When the price level of capital assets is high relative to the price level of current output, conditions are favorable for investment; when the price level of capital assets is low relative to the price level of current output, then conditions are not favorable for investment, and a recession -- or a depression -- is indicated. (1986, 143)

"The theoretical argument of how investment is determined involves a comparison of the [demand] price of capital assets and [the supply price] of investment output. In a corporate capitalist economy with a stock exchange, the market's valuation of a firm's capital assets and market position substitutes for the [demand] price of capital assets" (1986, 186).

However, Minsky parts company over the determination of stock prices, believing that they are determined in accordance with Keynes' "beauty contest" theory rather than fundamental valuation theory. This opens the possibility of waves of speculation and pessimism that can drive equity prices up and down.

A comparison of B-T and Minsky shows that they share a common view about the q transmission mechanism with both believing that equity prices matter for investment. However, they have different views about the rationality of the stock market as a regulator of capital accumulation. Both believe that equity prices value the existing capital stock, and both believe equity prices impact the "incentive" to invest through a cost of capital channel. They differ regarding the equity valuation process.

B-T adopt a fundamental valuation approach, while Minsky adopts a Keynesian beauty contest approach. Thus, it is not the causal logic of q that is at issue, but rather the social welfare significance of the economic signals provided by equity markets. For B-T, equity markets provide generally coherent signals directing investment and the accumulation of capital, and fluctuations in q provide signals about both shareholder required rates of return and the marginal efficiency of capital. For Minsky, these signals are dangerously contaminated with speculative noise, and they need contain no information about the marginal efficiency of capital.5

5. B-T do not believe that stock markets are always efficient (defined as equity prices equal to
Hayashi: neo-classical q.

If payments to capital are identified with the marginal product of capital, q can be given a neo-classical microeconomic interpretation whereby it is equivalent to the shadow value of an additional unit of installed capital. The equilibrium shadow value depends on the objective function of the firm and the competitive structure of the economy. If the firm is a profit maximizer and operates in a competitive economy, the equilibrium value is \( q = 1 \). If it is an imperfectly competitive profit maximizer, then the equilibrium value is \( q > 1 \). Lastly, as is shown in the next section, if the firm is a sales maximizer then the equilibrium value is \( q < 1 \).

Under the assumption that payments to capital equal the marginal product of capital, the net present value of a project becomes

\[
(10) \text{NPV} = P_K f_K / r_E - P_K > 0
\]

where \( f_K \) = marginal product of capital. \( P_K \) is the price of capital which equals the price of output since there is assumed to be a single good. Substituting (7) into (10), assuming \( V^E = V \), and manipulating yields

\[
(11) \text{NPV} = \left[ P_K f_K K / V \right] \left[ P_E / P_K K \right] - 1 > 0
\]

According to marginal productivity theory, profit is equal to the marginal product of capital times the level of capital, so that \( V = P_K f_K K \). Thus (11) can be expressed as

fundamental values). “Like Keynes, we believe that the stock market does not grind out values by mirroring the rational optimizations of informed managements but generates values of its own (B - T, 1990, p.548).” However, they do believe that investors value equities using a discounted dividend model, and that the demand for equities represents an implicit demand for capital. Unfortunately, since the discounted dividend model can produce tautologically any value of equities by appropriate choice of discount rate and dividend expectations, this blurs the distinction with Minsky. B-T are closer to the efficient market viewpoint, both with regard to the technical description of how stock prices are determined and the pattern of deviations from market efficiency. A “super strong” q can be defined as the case where stock markets are also assumed to be efficient.
(12) \( \text{NPV} = q - 1 > 0 \)

In a neo-classical world the condition for undertaking an investment project is therefore also \( q > 1 \). Firms are in long run equilibrium when they hold their desired capital stock and have no further incentive to invest, which corresponds to a situation in which \( \text{NPV} = 0 \) and \( q = 1 \). If market conditions are non-competitive, then capital earns more than its marginal product \( (V > P_k f_K K) \). Using (11), this implies the equilibrium value of \( q \) is greater than one.

This neo-classical interpretation of \( q \) has been formalized by Hayashi (1982) in a model of a profit maximizing perfectly competitive firm optimally accumulating capital subject to costly adjustment of the capital stock. The firm's discount rate is the cost of capital. Within this framework, \( q \) then bears an interpretation as the shadow price of an additional unit of installed capital, this being the increment to NPV resulting from an additional unit.

The Hayashi model explicitly locates \( q \) theory within marginal productivity theory, and this distinguishes it from both the B-T and Minsky constructions of \( q \).\(^6\) A second feature of Hayashi's model is that it lacks explicit recognition of the stock market. However, if firms' discount factor is assumed to be the cost of equity capital, this establishes a link with the stock market that has firms accumulating capital on the terms laid down by their shareholders.

At any moment in time, firms' \( q \) is endogenous and given by

\[
(13) \quad q_t = q(K_t, r_E, Z)
\]

It depends negatively on the current capital stock and firms' discount factor. The variable \( Z \)

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\(^6\) This means that returns to capital are technologically determined by the production function. However, since the marginal product of capital depends positively on the level of employment, fluctuations in employment caused by fluctuations in aggregate demand can still affect \( q \) and the level of investment. Thus, animal spirits can continue to matter, and if firms become pessimistic about future aggregate demand and employment conditions, this will lower both the expected future marginal product of capital and \( q \).
captures the effect of managers' goals and time horizons, and reflects the fact that $q$ is a shadow value that is endogenously determined as part of managers' maximization program. The value of $q$ will vary according to whether managers maximize profits or another goal such as firm size.

V The implications of owner-manager conflict and non-profit maximization for $q$ theory

Crotty (1990) accuses B-T of conflating owners and managers, thereby ignoring the essential presence of owner-manager conflict. This section shows that owner-manager conflict does not necessarily undermine the internal logic of $q$; however, it does change the equilibrium value of $q$.

The NPV calculation given by equation (8) is the heart of the firm's marginal investment decision. This equation assumes that firms use the cost of equity capital to discount future revenue streams. The Modigliani-Miller theorem maintains that the value of the firm is independent of the equity-debt mix, and that the cost of equity capital is the same as that of debt. If the Modigliani-Miller theorem is rejected, the cost of capital is a weighted average of the costs of equity and debt capital, with the weights depending on firms' debt-equity ratios. In this event, the effect of changes in the cost of equity on the cost of capital is diluted. However, the stock market and $q$ still matter, although the magnitude of influence is greatly lessened.

Proposition 1. If firms, for behavioral reasons, pay no heed to the cost of equity capital as determined by equity prices, then the $q$ channel is voided.

In this event, firms use a discount rate $r = r_E$ that is independent of the stock market, and the investment condition becomes

\[(8') \text{NPV} = \frac{R}{r} - P_K > 0\]

The investment decision is independent of equity prices.\(^7\) Firms are still maximizing profits in

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\(^7\)Investment spending and equity prices may still be "statistically" related even though they are not "causally" related. One reason is firms' and shareholders expectations of profits may move together. A second reason is that firms may use the bond rate as their cost of capital, and
the sense of maximizing the gap between revenues and costs, but they no longer use
shareholders' required rate of return (as reflected in the equity cost of capital) to discount
revenues and costs. This means managers do not maximize profits from "shareholders' point of
view", and it also breaks the $q$ channel.

**Proposition 2.** If firms use the equity cost of capital but are size maximizers rather than profit
maximizers, then equilibrium $q$ will be less than unity.

By pushing NPV equal to zero, equation (8) implicitly has firms maximizing profits. If they use
shareholders' discount rate, they maximize profits from shareholders' point of view: if they use
their own discount rate, they maximize profits from managers' point of view. An earlier
microeconomic literature on the firm (Archibald, 1971) argued that firms may maximize sales
subject to an institutionally determined minimum profit constraint. In this event, equation (8)
takes the form

$$(8'') \text{NPV} = \frac{R}{r_E} - P_K < -z$$

where $z = \text{reciprocal of the Lagrange multiplier associated with the minimum profit constraint.}$

This condition is derived in the appendix. Equation (8'') has firms pushing capital accumulation
beyond the point of profit maximization so that NPV on the marginal unit of capital is negative.

Just how negative it is depends on the minimum profit constraint: the lower this constraint, the
more negative the NPV.

Substituting for $r_E = V^E/P_E$ and setting $R = V/K = V^E/K$ yields

$$(14) \text{NPV} = q - 1 > -z/P_K$$

Thus, the effect of non-profit maximizing behavior is not to eliminate $q$, but rather to change the
equilibrium value. When perfectly competitive firms profit maximize, the equilibrium value is $q$

movements in bond rates affect the price portfolio holders are willing to pay for equities.
When they do not, the equilibrium value is \( q = 1 - \frac{z}{P_K} < 1 \). An empirically low observed \( q \) may therefore indicate non-profit maximizing behavior by firms. However, even though firms don't profit maximize, the influence of the stock market is still felt through the equity cost of capital, and a \( q \) channel is still operative. As with profit maximizing firms, it is only if non-profit maximizing firms don't use shareholders' cost of capital that the \( q \) channel disappears.

**Proposition 3.** If firms and shareholders have systematically different expectations regarding the stream of future earnings, equilibrium \( q \) will differ from unity. If shareholders are more optimistic, it will be greater than unity: if firms are more optimistic it will be less than unity.

In deriving \( q \), B-T assume that managers and shareholders holding the same profit expectations. If these expectations differ, so that \( V = V^E \), this yields the following NPV condition:

\[
NPV = \frac{R}{r_E} - P_K > 0 \\
= \frac{[V/K]}{[V^E/P_E^E]} - P_K > 0 \\
= \frac{[V/V^E]}{q} - 1 > 0
\]

The condition for equilibrium is no longer \( q = 1 \). Instead, \( q \) is weighted by the ratio of manager to shareholder profit expectations. If shareholders' are relatively more optimistic \( (V/V^E < 1) \), equilibrium stock market \( q \) will be greater than unity. The logic is that shareholders push up equity prices excessively, but managers who know the true value of profit streams are unwilling to invest at a level consonant with shareholder expectations. If firms are relatively more optimistic \( (V/V^E > 1) \), equilibrium stock market \( q \) is less than unity.

**VI Conclusion**

The spread of \( q \) theory within the economics profession has raised the policy salience of equity markets and equity prices. \( q \) theory has continuities with earlier Keynesian investment theory, and it also breaks with it. The continuities concern the fact that it remains a theory of
investment based upon the marginal efficiency of investment and the cost of capital. However, it emphasizes the cost of equity capital in place of the bond market interest rate. In the strong version, it also has equity markets sending coherent signals about the marginal efficiency of investment.

$q$ theory requires that managers use the equity cost of capital in making their investment decisions. Absent this, the stock market is irrelevant for investment. The equilibrium value of $q$ depends on the microeconomic description of the firm's objective function and competitive conditions. The strong version of $q$ has stock markets communicating information about the marginal efficiency of capital, which requires that stock markets value equities in accordance with the fundamental valuation model. If stock prices are determined in accordance with Keynes's beauty contest, they carry no information about the marginal efficiency of investment.

Ultimately, the relation between the stock market and investment is an empirical matter. On this score, the evidence suggests that after controlling for cyclical economic conditions, the stock market is a sideshow. Morck et al. (1990) find that a stock market variable is statistically significant in firm level investment equations, but the amount of investment that is explained is very small. Blanchard et al. (1993) find a similarly small role once investment fundamentals are included in aggregate investment equations. von Furstenberg (1977) concludes that including a distributed lag of $q_t$ in quarterly regressions containing capacity utilization, capital stock, and taxes, “must be regarded as optional” (p.388). Chirinko (1993) provides a complete survey of the literature on business fixed investment, including a survey of the empirical literature on $q$. He reports that $q$-theoretic empirical investment equations generate low $R^2$s and have substantial

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8. Technically, Morck et al. (1990) find that the stock market is incrementally unimportant for investment. This is because their equations are specified as first differences. However, empirical work in levels of investment (Clark, 1979) reports similar results.
residual serial correlation problems. Most importantly, they imply implausibly long paths of capital stock adjustment in response to disturbances to the optimal capital stock.

This raises questions as to why $q$ has proven so popular amongst economists. A speculative answer is that "strong" $q$ and "neo-classical" $q$ have a logical consistency with the Arrow - Debreu general equilibrium model that is the benchmark of modern economics. Neo-classical $q$ is consistent with marginal productivity theory of income distribution, while strong $q$ has managerially controlled firms accumulating capital in accordance with the wishes of rational shareholders as communicated via the stock market.

Finally, returning to the public policy discussion that opened the paper, if stock markets are inefficient (Shiller, 1981) and do not matter for investment (i.e $q$ has little impact), then their significance for economic growth is greatly reduced. If they are inefficient and do matter for investment, then there is a policy imperative to actively reduce their excess volatility as it distorts the growth path.
Appendix

This appendix derives the first-order conditions governing firms' desired capital stock. These first order conditions are derived from a static maximization program in which the firm is free to choose its optimal capital stock, and the derived value of $q$ therefore represents the steady state equilibrium value. This contrasts with the value of $q$ derived in a dynamic programming context (Hyashi, 1982) where the value of $q$ represents the instantaneous shadow price of a unit of capital, and it changes along the adjustment path. In long run equilibrium, the value of $q$ is the same under both the dynamic and static programs.

**Case 1.** For a profit maximizing perfectly competitive firm that uses the equity cost of capital in its calculus, the maximization program is

(A.1) \[ \text{Max } V = \frac{R(K)}{r_E} - P_K K \quad \text{subject to } R_K > 0, \quad R_{KK} < 0 \]

where $V = \text{net present value of profits}$
$K = \text{capital stock}$
$R(K) = \text{perpetual stream of earnings from capital stock of } K$
$r_E = \text{cost of equity capital}$
$P_K = \text{price of capital}$

The first order condition is $R_K/r_E - P_K = 0$ which corresponds to equation (8) in the main text. This first order condition can then be used to derive the equilibrium value of $q$ as is done in equation (9) in the text.

**Case 2.** For a profit maximizing firm that uses managers' own cost of capital instead of the equity cost of capital in its calculus, the maximization program is

(A.2) \[ \text{Max } V = \frac{R(K)}{r} - P_K K \quad \text{subject to } R_K > 0, \quad R_{KK} < 0 \]

where $r = \text{managers' cost of equity capital}$
$P_K = \text{price of capital}$

The first order condition is $R_K/r - P_K = 0$ which corresponds to equation (8') in the main text.

**Case 3.** For a size maximizing firm that uses the equity cost of capital and is constrained by a binding minimum profit constraint, the maximization program is

(A.2) \[ \text{Max } V = K \quad \text{subject to } \frac{R(K)}{r_E} - P_K K = V_{\text{MIN}} \]

where $V_{\text{MIN}} = \text{minimum allowable profit}$
The first order condition is $\frac{R_K}{r_E} - P_K = -z$ where $z$ is the reciprocal of the Lagrange multiplier. This expression corresponds to equation (8") in the main text.
References


Figure 1 Shows the component factors affecting the determination of $q$. 

$d(t)$

$r_E$  Shareholders

$V^E(t)$

$q$

$R$  Managers