Walras' Law and Keynesian Macroeconomics

Abstract

This paper examines the claim that Keynesian models violate Walras' law. Walras' law is founded in the logic of exchange. Standard statements misrepresent it, as it pertains to a monetary economy. Keynesian models are consistent with Walras' law once this misrepresentation is corrected. The law holds for both notional and effective demands. It also holds in unconstrained Walrasian equilibria, constrained Walrasian equilibria, and constrained non-Walrasian equilibria. The latter corresponds to a Keynesian conception of equilibrium: markets need not clear, but agents expectations must be fulfilled.

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Thomas I. Palley
Public Policy Dept., AFL-CIO
815 Sixteenth Street, NW
Washington, DC 20006

I Introduction

Walras' law is a critical component of economic theory. It derives from the nature of market exchange. As long as market exchange is the proceedure governing transactions between individual agents and firms, then Walras' law must hold. This is true independent of theoretical paradigm, be it Keynesian, Post Keynesian, classical, or new classical.

It is often claimed (Sargent, 1979, p.67 - 70) that Keynesian macroeconomic models violate Walras' law. The current paper seeks to explore this contention, and examines the place of Walras' law in Keynesian economics. Though highly abstract, this question is important: if Keynesian models violate Walras' law, this would seriously undermine their plausability. Similarly, if Keynesian models satisfactorily account for Walras' law, this lends credence to their theoretical foundations.

The place and specification of Walras' law has also been questioned in connection with Clower's (1965) theory of effective demand, and here the issue is how the introduction of effective demands changes the specification of Walras' law both in and out of equilibrium. These latter issues have been raised in an exchange between Rhodes (1984, 1991) and Greenfield (1986, 1991).

The current paper builds upon this exchange. The argument is that the confusion surrounding the place of Walras' law in Keynesian macroeconomics stems from misunderstanding of its analytic foundation, combined with its misrepresentation as it pertains to monetary economies. Once this misrepresentation is corrected, the focus shifts from concern with the place and specification of Walras' law, to concern with definition of an appropriate equilibrium concept.

The structure of the paper is as follows. Section II discusses the foundation of Walras' law and its relation to the process of exchange. Section III discusses the place of Walras' law in the neo-Keynesian ISLM model. Section IV then discusses how conventional statements of Walras' law

misrepresent the exchange process, and this misrepresentation explains why Keynesian models appear to violate Walras' law. Section V introduces Clower's (1965) distinction between "notional" and "effective" demands, and restates Walras' law in terms of effective demands. Lastly, section VI explores the relation between Walras' law and equilibrium in the Keynesian model. This necessitates distinguishing between "Walrasian" and "non-Walrasian" constructions of equilibrium. Keynesian macro models satisfy Walras' law while embodying a non-Walrasian equilibrium concept.

II Exchange as the foundation of Walras' law.

The foundation of Walras' law is the two sided nature of exchange. Decisions to purchase represent acts of demand, and involve buyers handing over something of equal monetary value to the seller. This handing over implicitly corresponds to an act of supply. Thus, for every act of demand there is an equal act of supply. Analogously, decisions to sell represent acts of supply, and involve a willingness to accept something of equal monetary value in exchange. This willingness to accept corresponds to an implicit act of demand. Thus, for every act of supply there is an equal act of demand. Acts of demand and supply therefore implicitly involve matching equal valued acts of supply and demand respectively. It is this correspondence that constitutes the foundation of Walras' law.

In a monetary economy, as in an indirect barter economy, the implicit act of demand that matches an act of supply must be interpreted carefully. This is because an agent selling a good or service may be willing to accept money or some good which they do not intend to keep. In this case there is a temporal aspect to exchange: in the first instance the supplier displays a "willingness to

accept" money (or a good in the case of indirect barter), and the money (good) is then re-traded for the item that is ultimately demanded.

The above explanation of Walras' law can be formalized in both one period and multi-period economies. If money is the medium of exchange, then the economy must of necessity be a multi-period economy. This is because agents in a single period economy would be unwilling to accept money in exchange as they would end up holding something with no "use" value at the period end.

Barter exchange

Within a one period barter economy with two agents (j = 1,2) and two goods (i = 1,2), Walras' law can be formalized as follows. Agent 1 has an endowment of good 1, while agent 2 has an endowment of good 2, and both agents desire to consume some of both goods. Agents take prices as exogenously given. When forming their plans, all agents assume they will be able to buy and sell as much as they wish at the announced prices. Walras' law says nothing about how prices are set, or the calculus of agents' decision making: it is simply a relation that follows from the process of exchange.

If agent 1 wishes to purchase some of good 2, she must offer an amount of good 1 of equal value in exchange which implies

(1) $p_1 S_{1,1} = p_2 D_{2,1}$

where $S_{i,j}$ = supply of good i by agent j

$D_{i,j}$ = demand for good i by agent j

For agent 2 there exists an analogous relation given by

(2) $p_2 S_{2,2} = p_1 D_{1,2}$

Summming equations (1) and (2) then yields

(3) $p_1(S_{1,1} - D_{1,2}) + p_2(S_{2,2} - D_{2,1}) = 0$

Equation (3) provides a statement of Walras' law for a two agent - two good barter economy. Together, equations (1) - (3) make it clear that the requirement of equal nominal exchange means that Walras' law must apply in any economy in which this requirement obtains.

Monetary exchange

A similar relation can be obtained for a monetary economy in which producers offer money in exchange for labor, and households offer money in exchange for output. In the labor market, firms who demand labor must offer money, and they are therefore bound by the following relation

(4) $wL_d = M_{sf,L}$

where w = nominal wage

L_d = firms' labor demand

$M_{sf,L}$ = money payment offerred by firms to suppliers of labor

Analogously, for households selling labor the relation is

(5) $wL_s = M_{dh,L}$

where L_s = labor supply

$M_{dh,L}$ = money payment demanded by households in return for labor

In the product market the relations are

(6) $py_s = M_{df,y}$

(7) $py_d = M_{sh,y}$

where p = price of output

y_s = supply of output

$M_{df,y}$ = money payment demanded by firms in return for output

y_d = demand for output

$M_{sh,y}$ = money payment offered by households in return for output

Subtracting (4) and (6) from (5) and (7) then yields

(8) p(yd - ys) + w(Ls - Ld) + (Mdf,y + Mdh,L - Msh,y - Msf,L) = 0

This is a statement of Walras Law for an economy with a single good, labor, and money. Walras' Law follows from the fact that all decisions to buy (demand) and sell (supply) generate matching supplies and demands of the medium of exchange.

The implication is clear: Walras' law must hold in any voluntary exchange economy, both in and out of equilibrium. Moreover, the law has nothing to do with the way in which prices are formed, the nature of agents' decision calculus, and the nature of equilibrium. These issues are separable from the exchange accounting relation that is Walras' law.

III ISLM and Walras' law

The ISLM model represents the conventional neo-Keynesian macro model. Within the model, points on the IS curve correspond to positions of goods market equilibrium, and imply that

(9) ys = yd

Analogously, points on the LM schedule correspond to positions of money market equilibrium. This equilibrium is obtained as part of overall portfolio equilibrium in which agents willingly hold the existing stock of wealth. The supplies and demands for wealth are given by

(10) Ws = Bs + Ms

(11) Wd = Bd + Md

while the portfolio equilibrium condition is

(12) Ws = Wd

Substituting (10) and (11) into (12), summing with (9), and rearranging yields

(13) (yd - ys) + (Bd - Bs) + (Md - Ms) = 0

This condition represents the conventional statement of Walras' law in the ISLM model. When economic outcomes lie at the intersection of the IS and LM schedules, this implies that yd = ys and Md = Ms. Combining with (13) then implies that Bd = Bs, and Walras' law is therefore claimed to be satisfied.

However, such a treatment ignores the labor market. Inclusion of the labor market results in a restatement of (13) given by

(13') (yd - ys) + (Bd - Bs) + (Md - Ms) + w(Nd - Ns) = 0

where w = real wage with prices normalized at unity. The ISLM claim that (yd - ys) = (Bd - Bs) = (Md - Ms) = 0, combined with the existence of unemployment which implies that (Nd - Ns) < 0, is inconsistent with equation (13'). It is this inconsistency that leads to the claim that the Keynesian macro model violates Walras' law.

IV The misrepresentation of monetary transacting

Section III reveals how Walras' law is apparently violated in the standard Keynesian ISLM. This section argues that this violation follows from the Keynesian model's misrepresentation of the role of money in the exchange process.

In a monetary economy, every transaction involves a transfer of money. As observed in section II, demand for goods is accompanied by an offer of money, while supply of goods is accompanied by a willingness to accept money. Similarly, in labor markets, demand for labor is accompanied by the offer of a money wage payment, while the supply of labor is accompanied by a willingness to accept a money wage payment. It is the failure to recognize these considerations that explains why Walras' law is violated in the Keynesian model. Once they are included, this gives rise to the following restatement of Walras' law

(14a) (yd - ys) + (Bd - Bs) + (Md + Dd + Wd - Ms - Ds - Ws)

+ w(Nd - Ns) = 0

(14b) Dd = ys

(14c) Ds = yd

(14d) Wd = wNs

(14e) Ws = wNd

where Dd = real value of money balances received for supplying good

Ds = real value of money balances offered in demand for goods

Wd = real value of money income demanded for supplying labor

Ws = real value of money paid in return for labor (the wage bill)

Md = portfolio demand for money

Ms = portfolio supply of money

In equation (14a) all variables are deflated by the price level. The critical innovation in this equation is the recognition that money transfers accompany every goods and labor market transaction. The monetary dimension of goods market transacting is captured by equations (14b) and (14c), while the monetary dimension of labor market transacting is captured by equations (14d) and (14e).

Once the monetary dimension of transacting is incorporated in Walras' law, it is simple to show that Walras' law holds for the Keynesian model. Points of intersection of the IS and LM schedules correspond to market clearing in the goods, bond and portfolio money markets, so that (yd - ys) = (Bd - Bs) = (Md - Ms) = (Dd - Ds) = 0. At the same time there is unemployment which implies w(Nd - Ns) < 0. Substituting in equation (14a) yields

(15) (Wd - Ws) + w(Nd - Ns) = 0

where $w(N_d - N_s) < 0$ and $(W_d - W_s) > 0$. Unemployment therefore corresponds to a situation of excess demand for money income, while excess demand for labor corresponds to a situation of excess supply of money income. This is the logic behind Greenfield's (1986, p.259) claim that effective excess supplies have as their reverse effective excess demand for money.

Recognizing the monetary dimension to transacting restores consistency of the Keynesian model with Walras' law. A second question then becomes whether a situation such as that represented by equation (15) corresponds to an equilibrium, or whether the situation is one of temporary disequilibrium. Resolving this question calls for distinguishing between (i) notional and effective demands, and (ii) Walrasian and non-Walrasian equilibria.

IV Notional demands, effective demands, and Walras' law

The standard derivation of the output, money, and bond supply and demand schedules in the Keynesian model is based on a Walrasian construction of markets in which buyers and sellers take prices as given and assume that they can buy and sell as much as they want to at those prices. Clower (1965) labelled demand and supply schedules derived under this set of assumptions as "notional" demand and supply schedules. Schedules derived in this fashion represent what buyers and sellers would like to buy and sell given that they are unconstrained in the quantities they can buy and sell at the announced price.

In place of the concept of notional demand and supply schedules, Clower introduced the concept of "effective" demand and supply schedules. These represent what buyers and sellers wish to buy and sell given that they recognize that they face constraints on the quantities they can buy and sell at existing prices. The imposition of these quantity constraints or quantity rations changes the

maximization program that buyers and sellers solve, and results in effective demand and supply schedules that differ from notional demand and supply schedules. These schedules are given by

(14) $zd^{\wedge} = d(r, p, x| Qe = 0) = zd = d(r, p, x| Qe = 0)$

(15) $zs^{\wedge} = s(r, p, x| Qe = 0) = zs = s(r, p, x| Qe = 0)$

where $zd^{\wedge}$ = vector of effective demands

zd = vector of notional demands

$zs^{\wedge}$ = vector of effective supplies

zs = vector of notional supplies

r = interest rate

p = vector of prices

x = vector of endowments

Qe = vector of expected quantity constraints

Just as for the case of notional demands and supplies, Walras' Law will continue to hold for the case of effective demands and supplies. This is because the logic of exchange continues to hold, which implies that a decision to buy involves an offer that is an act of supply, while a decision to supply involves a willingness to accept payment that is an act of demand. Consequently, Walras' law holds for economies in which agents face quantity constraints.

In a quantity constrained Keynesian macro model, Walras' law can therefore be written as

(16) $(yd^{\wedge} - ys^{\wedge}) + (Bd^{\wedge} - Bs) + (Md^{\wedge} + Dd^{\wedge} + Wd^{\wedge} - Ms - Ds^{\wedge} - Ws^{\wedge})$

$+ w(Nd^{\wedge} - Ns^{\wedge}) = 0$

In the ISLM model the goods, bond, and portfolio money markets continue to clear. However, the existence of involuntary unemployment implies $w(Nd^{\wedge} - Ns^{\wedge}) < 0$, which in turn implies $Wd^{\wedge} -$

$Ws\hat{} > 0$. Thus, the quantity constrained effective demand representation of Walras' law in the Keynesian model becomes

(17a) $(yd\hat{} - ys\hat{}) + (Bd\hat{} - Bs) + (Md\hat{} + Dd\hat{} + Wd\hat{} - Ms - Ds\hat{} - Ws\hat{})$

$+ w(Nd\hat{} - Ns\hat{}) = 0$

(17b) $(yd\hat{} - ys\hat{}) = (Bd\hat{} - Bs) = (Md\hat{} - Ms) = (Dd\hat{} - Ds\hat{}) = 0$

(17c) $Wd\hat{} - Ws\hat{} > 0$

(17d) $w(Nd\hat{} - Ns\hat{}) < 0$

V Walrasian versus non-Walrasian eqilibria

The introduction of effective demand and supply schedules in place of notional schedules changes the specifications of the functions embedded in Walras' law. However, the issue of equilibrium remains open. Resolving this issue requires distinguishing between Walrasian and non-Walrasian equilibria. The logic of exchange means that Walras' law applies in both types of equilibrium: what distinguishes the two is the characterization of equilibrium.

Equilibrium is a situation in which there is no force making for change. A Walrasian equilibrium is a particular type of equilibrium in which demand equals supply in each and every market, and agents' expectations are fulfilled. This latter requirement is important because the fulfillment of agents' expectations means that, given unchanged exogenous conditions, they will have no incentive to change their actions from period to period. Walrasian equilibria can in turn be divided into "unconstrained Walrasian equilibria" and "constrained Walrasian equilibria".

Unconstrained Walrasian equilibrim

An unconstrained Walrasian equilibrium is defined as a situation in which

(18a) $zd = d(r, p, x| Qe = 0)$

(18b) $zs = s(r, p, x| Qe = 0)$

(18c) $zd = zs$

(18d) $Qe = Q = 0$

where zd = vector of notional demands

zs = vector of notional supplies

r = interest rate

p = vector of prices

x = vector of endowments

Qe = vector of expected quantity constraints

Q = vector of actual constraints

The market supply and demand schedules correspond to notional demand an supply schedules in which agents anticipate zero quantity constraints. Each and every market clears, Walras' law holds, and expectations are fulfilled so that agents are indeed unconstrained.

Applied to macroeconomics, unconstrained Walrasian equilibrium implies

(19a) $(yd - ys) + (Bd - Bs) + (Md + Dd + Wd - Ms - Ds - Ws) + w(Nd - Ns) = 0$

(19b) $Ds = yd$

(19c) $Dd = ys$

(19d) $wNs = Wd$

(19e) $wNd = Ws$

(19f) $(yd - ys) = (Bd - Bs) = (Md - Ms) = (Dd - Ds)$

= (Wd - Ws) = w(Nd - Ns) = 0

(19g) Qe = Q = 0

This is the description of equilibrium that is associated with the classical macro model (Sargent, 1979).

Constrained Walrasian equilibrim

A constrained Walrasian equilibrium is defined as a situation in which

(20a) zd^ = d(r, p, x| Qe = 0)

(20b) zs^ = s(r, p, x| Qe = 0)

(20c) zd^ = zs^

(20d) Qe = Q = 0

where zd^ = vector of effective demands

zs^ = vector of effective supplies

As with an unconstrained Walrasian equilibrium, each and every market clears and agents' expectations of constraints are fulfilled. The key difference between the unconstrained and constrained Walrasian equilibrium is that market supply and demand schedules are effective demands and supplies in the latter. As a result, the theory of effective demands and supplies is relevant for Walrasian economic analysis.

Applied to macroeconomics, constrained Walrasian equilibrium implies

(21a) (yd^ - ys^) + (Bd^ - Bs) + (Md^ + Dd^ + Wd^ - Ms - Ds^ - Ws^)

+ w(Nd^ - Ns^) = 0

(21b) Ds^ = yd^

(21c) Dd^ = ys^

(21d) $wNs^ = Wd^$

(21e) $wNd^ = Ws^$

(21f) $(yd^ - ys^) = (Bd^ - Bs) = (Md^ - Ms) = (Dd^ - Ds^)$

$= (Wd^ - Ws^) = w(Nd^ - Ns^) = 0$

(21g) $Qe = Q = 0$

Constrained non-Walrasian equilibrium

The final class of equilibrium is that of constrained non-Walrasian equilibrium. The critical feature of non-Walrasian equilibria is that there is no requirement that markets clear. Consequently, some markets may clear while others do not. However, agents expectations must be fulfilled, or else agents would have an incentive to change their actions in ensuing periods. A constrained non-Walrasian equilibrium is defined as a situation in which

(22a) $zd^ = d(r, p, x| Qe = 0)$

(22b) $zs^ = s(r, p, x| Qe = 0)$

(22c) $zd^ >< zs^$

(22d) $Qe = Q = 0$

where the sign >< denotes may be greater than, less than, or equal.

The Keynesian ISLM model is intended to fit this category, and equilibrium in the Keynesian model is described by

(23a) $(yd^ - ys^) + (Bd^ - Bs) + (Md^ + Dd^ + Wd^ - Ms - Ds^ - Ws^)$

$+ w(Nd^ - Ns^) = 0$

(23b) $Ds^ = yd^$

(23c) $Dd^ = ys^$

(23d) (yd^ - ys^) = (Bd^ - Bs) = (Md^ - Ms) = (Dd^ - Ds^) = 0

(23e) (Wd^ - Ws^) - w(Nd^ - Ns^) = 0

(23f) (Wd^ - Ws^) > 0

(23g) w(Nd^ - Ns^) < 0

(23h) Qe = Q = 0

Within such an equilibrium, Walras' law holds. The Goods, bond, and portfolio money markets all clear, but there is involuntary unemployment that is matched by an excess demand for wage income. Market demand and supply schedules are effective demands and supplies, and actual outcomes confirm the expected constraints on which agents formed their effective demands and supplies. As a result, agents have no incentive to change the effective demands and supplies that gave rise to the equilibrium.

VI Some further issues

The above taxonomy of equilibria serves to define and highlight a range of issues. One long standing question has been the place of Walras' law in macroeconomics and whether Keynesian macro models satisfy Walras' law. The analysis has shown that Walras' law applies to all models, including the Keynesian model. However, Walras' law has nothing to do with equilibrium; instead, it is an accounting identity that follows from the logic of exchange, and which holds in and out of equilibrium.

The taxonomy serves to define a potential universe of types of equilibrium, but questions of existence and stability have not been addressed. It is also noteworthy that different theoretical paradigms may use similar equilibrium concepts. Thus, traditional Keynesian macroeconomics, as represented by the ISLM model, uses a constrained non-Walrasian concept of equilibrium.

However, new Keynesian models which make no use of the theory of aggregate effective demand also use this concept of equilibrium, as evidenced in efficiency wage models of unemployment (Shapiro and Stiglitz, 1984). Moreover, new Keynesian models also sometimes adopt a constrained Walrasian equilibrium concept. This is evident in the macroeconomic coordination literature (Cooper and John, 1988) in which the economy is characterized by multiple equilibria, and the economy can get stuck at low levels of activity because agents have self-fulfilling expectations that they will be constrained.

The general disequilibrium literature pioneered by Barro and Grossman (1971) has come to be identified with the Keynesian approach to constrained non-Walrasian equilibrium analysis. Unfortunately, this literature explicitly identifies itself with fix-price analysis. However, there is nothing in the Keynesian constrained non-Walrasian perspective that requires this assumption. Prices and nominal wages may be fully flexible, but given agents expectations of quantity constraints, the level of effective demand and supply can still settle at a level which confirms agents' expectations.

This possibility raises questions as to how prices are formed in markets where there is effective excess demand or supply. The traditional Walrasian approach to price adjustment is that prices respond to the existence of excess demands and supplies, and it is the continued application of this theoretical presumption that has led to the characterization of Keynesian macroeconomics as fix-price temporary disequilibrium analysis. An alternative to the Walrasian approach to price adjustment is the adoption of imperfect competition (see Benassy, 1993), in which case prices need not change despite the presence of effective excess supplies. A second alternative is to have prices adjusting to changes in the level of effective excess supplies and demands as follows

(24) DP = Dyd^ - Dys^

where D = first difference. In this case, as long as effective demands and supplies are unchanged across periods, prices would remain unchanged. In a Keynesian constrained non-Walrasian equilibrium excess effective demands and supplies could be non-zero, yet there would be no pressure for price change since agents expectations are fulfilled and effective excess demands and supplies are therefore unchanged.

References

Barro, R., and Grossman, H., "A General Disequilibrium Model of Income and Employment," American Economic Review, 80 (November/December 1971), 1095 - 1117.

Benassy, J.P., "Nonclearing Markets: Microeconomic Concepts and Macroeconomic Applications," Journal of Economic Literature, 31 (June 1993), 732-61.

Clower, R.(1965), "The Keynesian Counter-revolution: A Theoretical Appraisal," in F.H.Hahn and F.P.R.Brechling (eds.), The Theory of Interest Rates(London:MacMillan).

Cooper, R., and John, A., "Coordinating Coordination Failures in Keynesian Models," Quarterly Journal of Economics, 103 (August 1988), 441-63.

Greenfield, R.I.(1986), "Walras' Law in Macroeconomic Disequilibrium," Australian Economic Papers, vol. 25.

----------(1991), "On Disequilibrium Supply and Demand: Rejoinder to Rhodes," Australian Economic Papers, vol. 30.

Rhodes, J.R.(1984), "Walras' Law and Clower's Inequality," Australian Economic Papers, vol.23.

----------(1991), "Walras Law in Macroeconomic Disequilibrium: A Reply," Australian Economic Papers, vol. 30.

Sargent, T.J., Macroeconomic Theory, New York:Academic Press, 1979.

Shapiro, C., and Stiglitz, J.E., "Equilibrium Unemployment as a Worker Discipline Device," American Economic Review, 74 (June 1984), 433-44.

Varian, H.R.(1978), Microeconomic Analysis, New York:W.W.Norton & Co.

. The current treatment of Walras' Law in terms of the mechanics of exchange may be contrasted with the standard treatment which emphasizes the role of budget constraints (see Varian, p.140).
.Equation (13') can be made to hold by setting Ns equal to actual employment, Nd. However, this continues to violate Walras' law since the labor supply offers of the unemployed are not taken account of.
.In Keynesian models the stock of money and bonds is taken as exogenous. If they were endogenously determined there would be a need to distinguish between notional and effective supplies of money and bonds.